

UCLA-ALIGNED COURSE FIELD NOTES

STATS 10

Introduction to Statistical Reasoning

Study design, data summaries, uncertainty, inference, and regression for evidence-based reasoning

How these notes are written. I wrote each chapter the way I wish the material had been explained the first time: the real idea, the point where I usually get stuck, the simplest mental model that still stays honest, and a worked decision instead of a polished answer appearing from nowhere. Every chapter closes with practice and a fully written solution. If a sentence sounds impressive but does not help me solve or explain something, it does not belong here.

Daniel Mastick

Curriculum map, working notes, practice, and solutions

Course map

Week	Core thread	What should be solid by the end
1	Statistical questions, variables, and data stories	Reconstruct the model, work an application, test its assumptions, and explain the result in ordinary language.
2	Sampling, experiments, and causal claims	Reconstruct the model, work an application, test its assumptions, and explain the result in ordinary language.
3	Distributions, center, spread, and shape	Reconstruct the model, work an application, test its assumptions, and explain the result in ordinary language.
4	Relationships, correlation, and lurking variables	Reconstruct the model, work an application, test its assumptions, and explain the result in ordinary language.
5	Probability and simulation as long-run models	Reconstruct the model, work an application, test its assumptions, and explain the result in ordinary language.
6	Sampling distributions and standard error	Reconstruct the model, work an application, test its assumptions, and explain the result in ordinary language.
7	Confidence intervals and significance tests	Reconstruct the model, work an application, test its assumptions, and explain the result in ordinary language.
8	Simple regression, residuals, and responsible communication	Reconstruct the model, work an application, test its assumptions, and explain the result in ordinary language.
9	Integration studio	Combine several chapters in one case, preserve their assumptions, and reconcile the result across representations.
10	Cumulative review	Retrieve the full course map from memory and solve mixed problems without chapter labels.

Scope anchor: <https://catalog.registrar.ucla.edu/course/2025/STATS10>. The sequence is aligned to the official catalog description and expanded into a practical ten-week study plan.

How I would use this packet

Treat each numbered section as one chapter. Read the formal explanation, pause at the student interpretation, and see whether the easy version still says the same thing. Rework the example without looking, then cover the solution in the chapter practice box. The cumulative set at the end is deliberately mixed: knowledge becomes durable when I have to choose the tool instead of being told which chapter I am in.

Contents

1	Statistical questions, variables, and data stories	5
1.1	Rebuild the chapter from first principles	5
1.2	Details I would refuse to skip	5
1.3	How this chapter connects	6
1.4	The full working model	6
2	Sampling, experiments, and causal claims	7
2.1	Rebuild the chapter from first principles	7
2.2	Details I would refuse to skip	8
2.3	How this chapter connects	8
2.4	The full working model	8
3	Distributions, center, spread, and shape	10
3.1	Rebuild the chapter from first principles	10
3.2	Details I would refuse to skip	10
3.3	How this chapter connects	11
3.4	The full working model	11
4	Relationships, correlation, and lurking variables	12
4.1	Rebuild the chapter from first principles	12
4.2	Details I would refuse to skip	13
4.3	How this chapter connects	13
4.4	The full working model	14
5	Probability and simulation as long-run models	15
5.1	Rebuild the chapter from first principles	15
5.2	Details I would refuse to skip	16
5.3	How this chapter connects	16
5.4	The full working model	16
6	Sampling distributions and standard error	18
6.1	Rebuild the chapter from first principles	18
6.2	Details I would refuse to skip	18
6.3	How this chapter connects	19

6.4	The full working model	19
7	Confidence intervals and significance tests	20
7.1	Rebuild the chapter from first principles	20
7.2	Details I would refuse to skip	21
7.3	How this chapter connects	21
7.4	The full working model	21
8	Simple regression, residuals, and responsible communication	23
8.1	Rebuild the chapter from first principles	23
8.2	Details I would refuse to skip	23
8.3	How this chapter connects	24
8.4	The full working model	24

CHAPTER 1 Statistical questions, variables, and data stories

The idea

Statistics begins by translating a real question into observational units, variables, a target population, and a measurable claim. Categorical and quantitative variables support different summaries, while context determines whether a number is meaningful.

Rebuild the chapter from first principles

The claim I need to own is this: Statistics begins by translating a real question into observational units, variables, a target population, and a measurable claim. Categorical and quantitative variables support different summaries, while context determines whether a number is meaningful. I do not count that as learned just because the sentence looks familiar. I should be able to name the objects, state what changes and what stays fixed, recover the rule from the definitions, and explain why the result is allowed.

The move

My working sequence

1. **Translate.** Rewrite the prompt in the language of statistical questions, variables, and data stories. List the known quantities, the unknown, the units or types, and any hidden constraint.
2. **Choose.** State the theorem, model, algorithm, accounting rule, or physical law before substituting values. This is where I make the assumption visible.
3. **Execute.** Work one justified step at a time and keep intermediate state readable enough that I can audit it later.
4. **Verify.** Check a boundary case, sign, unit, invariant, order of magnitude, or alternative derivation. Then translate the result back into an ordinary sentence.

Details I would refuse to skip

- **Definitions:** I should be able to define every technical noun in the chapter without using that same noun in the definition.
- **Assumptions:** I write the condition that makes the method legal beside the method itself. A correct procedure under the wrong assumptions is still a wrong answer.
- **Representation:** I move freely among words, equations or code, a diagram, and a small numerical example. If one representation disagrees with another, that disagreement is useful evidence.
- **Failure modes:** I keep the nearest counterexample in mind: the boundary where the rule changes, the invariant breaks, or the model stops matching reality.
- **Communication:** My final answer names what the result means and what it does not establish. I do not let notation do the explanatory work for me.

Sanity check

Closed-note check. Can I teach statistical questions, variables, and data stories on a blank page, derive or reconstruct its main rule, work a fresh example, and diagnose a deliberately broken solution? If I can only recognize the finished work, I am not done.

How this chapter connects

I read this chapter as the bridge from the course foundations to sampling, experiments, and causal claims. The earlier material supplies the language and constraints; this chapter adds a new reasoning move; the later material asks me to use that move without being told its name. That connection matters because exams and real projects mix chapters on purpose.

The full working model

- **Core claim.** Statistics begins by translating a real question into observational units, variables, a target population, and a measurable claim.
- **What moves.** Categorical and quantitative variables support different summaries, while context determines whether a number is meaningful.
- **What keeps the answer honest.** The model becomes useful only when I can apply statistical questions, variables, and data stories to an unfamiliar case and defend the assumptions.
- **Mastery standard.** I can define the objects, rebuild the rule, work a fresh case, compare alternatives, and diagnose a broken solution without relying on recognition.

How I actually think about it

I write one sentence naming who, what, and why before I calculate anything; otherwise I can produce a clean answer to the wrong question.

The easy version

Statistics begins by translating a real question into observational units, variables, a target population, and a measurable claim.

Worked example

Question. A campus survey asks 200 library visitors how many hours they study. What population can it directly describe?

Walkthrough. It directly describes the sampled library visitors at that time. Generalizing to all UCLA students needs a sampling argument because non-visitors had no chance to enter the sample.

Common miss

The fastest way to get this chapter wrong is to use the visible procedure without naming its assumption, units, time period, reference group, or boundary. I put that condition beside the

work and verify the result independently.

Solved chapter practice

Practice 1. A campus survey asks 200 library visitors how many hours they study. What population can it directly describe?

Solution. It directly describes the sampled library visitors at that time. Generalizing to all UCLA students needs a sampling argument because non-visitors had no chance to enter the sample.

Practice 2. Explain statistical questions, variables, and data stories to a classmate using one definition, one concrete example, one assumption, and one failure mode.

Solution. Begin with the core claim above, use the worked case as the concrete example, state the condition highlighted by the warning, and choose a failure where that condition does not hold. A complete explanation connects all four pieces instead of listing them.

Mastery practice A. Reconstruct statistical questions, variables, and data stories from a blank page. Include the central definition, the governing rule, one required assumption, and one boundary case.

Solution. Start from the blue idea box, but rewrite it in my own words. Identify the objects and state, name the rule before using it, copy the relevant assumption from the details audit, and choose a boundary where that assumption becomes equality or fails. A complete solution explains why each part belongs; a list of vocabulary is not enough.

Mastery practice B. A classmate gets a polished-looking answer to a statistical questions, variables, and data stories problem but never states a model or checks the result. Write the shortest useful review of that solution.

Solution. I would ask four concrete questions: What exactly are the inputs and desired output? Which rule licenses the main step? Which assumption could invalidate it? Which independent check supports the conclusion? If the answer cannot survive those four questions, I treat the numerical or verbal result as unverified.

CHAPTER 2 Sampling, experiments, and causal claims

The idea

Random sampling supports population generalization, while random assignment supports causal comparison. Confounding, selection, nonresponse, placebo effects, blinding, replication, blocking, and paired designs determine what a study can honestly claim.

Rebuild the chapter from first principles

The claim I need to own is this: Random sampling supports population generalization, while random assignment supports causal comparison. Confounding, selection, nonresponse, placebo effects, blinding, replication, blocking, and paired designs determine what a study can honestly claim. I do not count that as learned just because the sentence looks familiar. I should be able to name the objects, state what changes and what stays fixed, recover the rule from the definitions, and explain why the result is allowed.

The move

My working sequence

1. **Translate.** Rewrite the prompt in the language of sampling, experiments, and causal claims. List the known quantities, the unknown, the units or types, and any hidden constraint.
2. **Choose.** State the theorem, model, algorithm, accounting rule, or physical law before substituting values. This is where I make the assumption visible.
3. **Execute.** Work one justified step at a time and keep intermediate state readable enough that I can audit it later.
4. **Verify.** Check a boundary case, sign, unit, invariant, order of magnitude, or alternative derivation. Then translate the result back into an ordinary sentence.

Details I would refuse to skip

- **Definitions:** I should be able to define every technical noun in the chapter without using that same noun in the definition.
- **Assumptions:** I write the condition that makes the method legal beside the method itself. A correct procedure under the wrong assumptions is still a wrong answer.
- **Representation:** I move freely among words, equations or code, a diagram, and a small numerical example. If one representation disagrees with another, that disagreement is useful evidence.
- **Failure modes:** I keep the nearest counterexample in mind: the boundary where the rule changes, the invariant breaks, or the model stops matching reality.
- **Communication:** My final answer names what the result means and what it does not establish. I do not let notation do the explanatory work for me.

Sanity check

Closed-note check. Can I teach sampling, experiments, and causal claims on a blank page, derive or reconstruct its main rule, work a fresh example, and diagnose a deliberately broken solution? If I can only recognize the finished work, I am not done.

How this chapter connects

I read this chapter as the bridge from statistical questions, variables, and data stories to distributions, center, spread, and shape. The earlier material supplies the language and constraints; this chapter adds a new reasoning move; the later material asks me to use that move without being told its name. That connection matters because exams and real projects mix chapters on purpose.

The full working model

- **Core claim.** Random sampling supports population generalization, while random assignment supports causal comparison.
- **What moves.** Confounding, selection, nonresponse, placebo effects, blinding, replication, blocking, and paired designs determine what a study can honestly claim.

- **What keeps the answer honest.** The model becomes useful only when I can apply sampling, experiments, and causal claims to an unfamiliar case and defend the assumptions.
- **Mastery standard.** I can define the objects, rebuild the rule, work a fresh case, compare alternatives, and diagnose a broken solution without relying on recognition.

How I actually think about it

Sampling answers who the result travels to; assignment answers whether the treatment caused the difference. I keep those jobs separate.

The easy version

Random sampling supports population generalization, while random assignment supports causal comparison.

Worked example

Question. Students choose whether to attend tutoring and attendees earn higher grades. Does the comparison prove tutoring caused the gain?

Walkthrough. No. Motivation, preparation, and schedule flexibility may affect both attendance and grades. A randomized offer or a defensible quasi-experiment would better isolate causation.

Common miss

The fastest way to get this chapter wrong is to use the visible procedure without naming its assumption, units, time period, reference group, or boundary. I put that condition beside the work and verify the result independently.

Solved chapter practice

Practice 1. Students choose whether to attend tutoring and attendees earn higher grades. Does the comparison prove tutoring caused the gain?

Solution. No. Motivation, preparation, and schedule flexibility may affect both attendance and grades. A randomized offer or a defensible quasi-experiment would better isolate causation.

Practice 2. Explain sampling, experiments, and causal claims to a classmate using one definition, one concrete example, one assumption, and one failure mode.

Solution. Begin with the core claim above, use the worked case as the concrete example, state the condition highlighted by the warning, and choose a failure where that condition does not hold. A complete explanation connects all four pieces instead of listing them.

Mastery practice A. Reconstruct sampling, experiments, and causal claims from a blank page. Include the central definition, the governing rule, one required assumption, and one boundary case.

Solution. Start from the blue idea box, but rewrite it in my own words. Identify the objects and state, name the rule before using it, copy the relevant assumption from the details audit, and choose a boundary where that assumption becomes equality or fails. A complete solution explains why each part belongs; a list of vocabulary is not enough.

Mastery practice B. A classmate gets a polished-looking answer to a sampling, experiments,

and causal claims problem but never states a model or checks the result. Write the shortest useful review of that solution.

Solution. I would ask four concrete questions: What exactly are the inputs and desired output? Which rule licenses the main step? Which assumption could invalidate it? Which independent check supports the conclusion? If the answer cannot survive those four questions, I treat the numerical or verbal result as unverified.

CHAPTER 3 Distributions, center, spread, and shape

The idea

A distribution is described by shape, center, spread, and unusual observations. Mean and standard deviation respond strongly to tails; median and interquartile range are resistant. Standardization puts values on a common distance-from-center scale.

Rebuild the chapter from first principles

The claim I need to own is this: A distribution is described by shape, center, spread, and unusual observations. Mean and standard deviation respond strongly to tails; median and interquartile range are resistant. Standardization puts values on a common distance-from-center scale. I do not count that as learned just because the sentence looks familiar. I should be able to name the objects, state what changes and what stays fixed, recover the rule from the definitions, and explain why the result is allowed.

The move

My working sequence

1. **Translate.** Rewrite the prompt in the language of distributions, center, spread, and shape. List the known quantities, the unknown, the units or types, and any hidden constraint.
2. **Choose.** State the theorem, model, algorithm, accounting rule, or physical law before substituting values. This is where I make the assumption visible.
3. **Execute.** Work one justified step at a time and keep intermediate state readable enough that I can audit it later.
4. **Verify.** Check a boundary case, sign, unit, invariant, order of magnitude, or alternative derivation. Then translate the result back into an ordinary sentence.

Details I would refuse to skip

- **Definitions:** I should be able to define every technical noun in the chapter without using that same noun in the definition.
- **Assumptions:** I write the condition that makes the method legal beside the method itself. A correct procedure under the wrong assumptions is still a wrong answer.

- **Representation:** I move freely among words, equations or code, a diagram, and a small numerical example. If one representation disagrees with another, that disagreement is useful evidence.
- **Failure modes:** I keep the nearest counterexample in mind: the boundary where the rule changes, the invariant breaks, or the model stops matching reality.
- **Communication:** My final answer names what the result means and what it does not establish. I do not let notation do the explanatory work for me.

Sanity check

Closed-note check. Can I teach distributions, center, spread, and shape on a blank page, derive or reconstruct its main rule, work a fresh example, and diagnose a deliberately broken solution? If I can only recognize the finished work, I am not done.

How this chapter connects

I read this chapter as the bridge from sampling, experiments, and causal claims to relationships, correlation, and lurking variables. The earlier material supplies the language and constraints; this chapter adds a new reasoning move; the later material asks me to use that move without being told its name. That connection matters because exams and real projects mix chapters on purpose.

The full working model

- **Core claim.** A distribution is described by shape, center, spread, and unusual observations.
- **What moves.** Mean and standard deviation respond strongly to tails; median and interquartile range are resistant.
- **What keeps the answer honest.** Standardization puts values on a common distance-from-center scale.
- **Mastery standard.** I can define the objects, rebuild the rule, work a fresh case, compare alternatives, and diagnose a broken solution without relying on recognition.

How I actually think about it

I sketch the distribution before choosing a summary because the shape tells me which number will lie to me least.

The easy version

A distribution is described by shape, center, spread, and unusual observations.

Worked example

Question. For 2, 3, 3, 4, and 18, compare the mean and median.

Walkthrough. The mean is 6 and the median is 3. The high outlier pulls the mean upward, so the median better represents a typical value in this small skewed set.

Common miss

The fastest way to get this chapter wrong is to use the visible procedure without naming its assumption, units, time period, reference group, or boundary. I put that condition beside the work and verify the result independently.

Solved chapter practice

Practice 1. For 2, 3, 3, 4, and 18, compare the mean and median.

Solution. The mean is 6 and the median is 3. The high outlier pulls the mean upward, so the median better represents a typical value in this small skewed set.

Practice 2. Explain distributions, center, spread, and shape to a classmate using one definition, one concrete example, one assumption, and one failure mode.

Solution. Begin with the core claim above, use the worked case as the concrete example, state the condition highlighted by the warning, and choose a failure where that condition does not hold. A complete explanation connects all four pieces instead of listing them.

Mastery practice A. Reconstruct distributions, center, spread, and shape from a blank page. Include the central definition, the governing rule, one required assumption, and one boundary case.

Solution. Start from the blue idea box, but rewrite it in my own words. Identify the objects and state, name the rule before using it, copy the relevant assumption from the details audit, and choose a boundary where that assumption becomes equality or fails. A complete solution explains why each part belongs; a list of vocabulary is not enough.

Mastery practice B. A classmate gets a polished-looking answer to a distributions, center, spread, and shape problem but never states a model or checks the result. Write the shortest useful review of that solution.

Solution. I would ask four concrete questions: What exactly are the inputs and desired output? Which rule licenses the main step? Which assumption could invalidate it? Which independent check supports the conclusion? If the answer cannot survive those four questions, I treat the numerical or verbal result as unverified.

CHAPTER 4 Relationships, correlation, and lurking variables

The idea

Scatterplots reveal direction, form, strength, and outliers. Correlation measures linear association on a unit-free scale but is sensitive to outliers, misses nonlinear structure, and does not establish causation.

Rebuild the chapter from first principles

The claim I need to own is this: Scatterplots reveal direction, form, strength, and outliers. Correlation measures linear association on a unit-free scale but is sensitive to outliers, misses nonlinear structure, and does not establish causation. I do not count that as learned just because the sentence looks familiar.

I should be able to name the objects, state what changes and what stays fixed, recover the rule from the definitions, and explain why the result is allowed.

The move

My working sequence

1. **Translate.** Rewrite the prompt in the language of relationships, correlation, and lurking variables. List the known quantities, the unknown, the units or types, and any hidden constraint.
2. **Choose.** State the theorem, model, algorithm, accounting rule, or physical law before substituting values. This is where I make the assumption visible.
3. **Execute.** Work one justified step at a time and keep intermediate state readable enough that I can audit it later.
4. **Verify.** Check a boundary case, sign, unit, invariant, order of magnitude, or alternative derivation. Then translate the result back into an ordinary sentence.

Details I would refuse to skip

- **Definitions:** I should be able to define every technical noun in the chapter without using that same noun in the definition.
- **Assumptions:** I write the condition that makes the method legal beside the method itself. A correct procedure under the wrong assumptions is still a wrong answer.
- **Representation:** I move freely among words, equations or code, a diagram, and a small numerical example. If one representation disagrees with another, that disagreement is useful evidence.
- **Failure modes:** I keep the nearest counterexample in mind: the boundary where the rule changes, the invariant breaks, or the model stops matching reality.
- **Communication:** My final answer names what the result means and what it does not establish. I do not let notation do the explanatory work for me.

Sanity check

Closed-note check. Can I teach relationships, correlation, and lurking variables on a blank page, derive or reconstruct its main rule, work a fresh example, and diagnose a deliberately broken solution? If I can only recognize the finished work, I am not done.

How this chapter connects

I read this chapter as the bridge from distributions, center, spread, and shape to probability and simulation as long-run models. The earlier material supplies the language and constraints; this chapter adds a new reasoning move; the later material asks me to use that move without being told its name. That connection matters because exams and real projects mix chapters on purpose.

The full working model

- **Core claim.** Scatterplots reveal direction, form, strength, and outliers.
- **What moves.** Correlation measures linear association on a unit-free scale but is sensitive to outliers, misses nonlinear structure, and does not establish causation.
- **What keeps the answer honest.** The model becomes useful only when I can apply relationships, correlation, and lurking variables to an unfamiliar case and defend the assumptions.
- **Mastery standard.** I can define the objects, rebuild the rule, work a fresh case, compare alternatives, and diagnose a broken solution without relying on recognition.

How I actually think about it

I refuse to read a correlation coefficient before seeing the plot; one number can hide curves, clusters, or a single influential point.

The easy version

Scatterplots reveal direction, form, strength, and outliers.

Worked example

Question. Ice-cream sales and drownings rise together in summer. Name a plausible lurking variable.

Walkthrough. Temperature or season affects both swimming exposure and ice-cream demand. The positive association does not imply that buying ice cream causes drownings.

Common miss

The fastest way to get this chapter wrong is to use the visible procedure without naming its assumption, units, time period, reference group, or boundary. I put that condition beside the work and verify the result independently.

Solved chapter practice

Practice 1. Ice-cream sales and drownings rise together in summer. Name a plausible lurking variable.

Solution. Temperature or season affects both swimming exposure and ice-cream demand. The positive association does not imply that buying ice cream causes drownings.

Practice 2. Explain relationships, correlation, and lurking variables to a classmate using one definition, one concrete example, one assumption, and one failure mode.

Solution. Begin with the core claim above, use the worked case as the concrete example, state the condition highlighted by the warning, and choose a failure where that condition does not hold. A complete explanation connects all four pieces instead of listing them.

Mastery practice A. Reconstruct relationships, correlation, and lurking variables from a blank page. Include the central definition, the governing rule, one required assumption, and one boundary case.

Solution. Start from the blue idea box, but rewrite it in my own words. Identify the objects and state, name the rule before using it, copy the relevant assumption from the details audit, and choose a boundary where that assumption becomes equality or fails. A complete solution explains why each part belongs; a list of vocabulary is not enough.

Mastery practice B. A classmate gets a polished-looking answer to a relationships, correlation, and lurking variables problem but never states a model or checks the result. Write the shortest useful review of that solution.

Solution. I would ask four concrete questions: What exactly are the inputs and desired output? Which rule licenses the main step? Which assumption could invalidate it? Which independent check supports the conclusion? If the answer cannot survive those four questions, I treat the numerical or verbal result as unverified.

CHAPTER 5 Probability and simulation as long-run models

The idea

Probability models uncertain processes through outcomes, events, conditional information, independence, and long-run frequency. Simulation approximates a probability by repeatedly reproducing the random mechanism, not by inventing arbitrary numbers.

Rebuild the chapter from first principles

The claim I need to own is this: Probability models uncertain processes through outcomes, events, conditional information, independence, and long-run frequency. Simulation approximates a probability by repeatedly reproducing the random mechanism, not by inventing arbitrary numbers. I do not count that as learned just because the sentence looks familiar. I should be able to name the objects, state what changes and what stays fixed, recover the rule from the definitions, and explain why the result is allowed.

The move

My working sequence

1. **Translate.** Rewrite the prompt in the language of probability and simulation as long-run models. List the known quantities, the unknown, the units or types, and any hidden constraint.
2. **Choose.** State the theorem, model, algorithm, accounting rule, or physical law before substituting values. This is where I make the assumption visible.
3. **Execute.** Work one justified step at a time and keep intermediate state readable enough that I can audit it later.
4. **Verify.** Check a boundary case, sign, unit, invariant, order of magnitude, or alternative derivation. Then translate the result back into an ordinary sentence.

Details I would refuse to skip

- **Definitions:** I should be able to define every technical noun in the chapter without using that same noun in the definition.
- **Assumptions:** I write the condition that makes the method legal beside the method itself. A correct procedure under the wrong assumptions is still a wrong answer.
- **Representation:** I move freely among words, equations or code, a diagram, and a small numerical example. If one representation disagrees with another, that disagreement is useful evidence.
- **Failure modes:** I keep the nearest counterexample in mind: the boundary where the rule changes, the invariant breaks, or the model stops matching reality.
- **Communication:** My final answer names what the result means and what it does not establish. I do not let notation do the explanatory work for me.

Sanity check

Closed-note check. Can I teach probability and simulation as long-run models on a blank page, derive or reconstruct its main rule, work a fresh example, and diagnose a deliberately broken solution? If I can only recognize the finished work, I am not done.

How this chapter connects

I read this chapter as the bridge from relationships, correlation, and lurking variables to sampling distributions and standard error. The earlier material supplies the language and constraints; this chapter adds a new reasoning move; the later material asks me to use that move without being told its name. That connection matters because exams and real projects mix chapters on purpose.

The full working model

- **Core claim.** Probability models uncertain processes through outcomes, events, conditional information, independence, and long-run frequency.
- **What moves.** Simulation approximates a probability by repeatedly reproducing the random mechanism, not by inventing arbitrary numbers.
- **What keeps the answer honest.** The model becomes useful only when I can apply probability and simulation as long-run models to an unfamiliar case and defend the assumptions.
- **Mastery standard.** I can define the objects, rebuild the rule, work a fresh case, compare alternatives, and diagnose a broken solution without relying on recognition.

How I actually think about it

When the formula feels opaque, I simulate the story and check whether the analytic answer has the right scale.

The easy version

Probability models uncertain processes through outcomes, events, conditional information, independence, and long-run frequency.

Worked example

Question. A fair coin is tossed twice. What is the probability of at least one head?

Walkthrough. The equally likely outcomes are HH, HT, TH, and TT. Three contain at least one head, so the probability is $3/4$; equivalently use one minus $P(TT)$.

Common miss

The fastest way to get this chapter wrong is to use the visible procedure without naming its assumption, units, time period, reference group, or boundary. I put that condition beside the work and verify the result independently.

Solved chapter practice

Practice 1. A fair coin is tossed twice. What is the probability of at least one head?

Solution. The equally likely outcomes are HH, HT, TH, and TT. Three contain at least one head, so the probability is $3/4$; equivalently use one minus $P(TT)$.

Practice 2. Explain probability and simulation as long-run models to a classmate using one definition, one concrete example, one assumption, and one failure mode.

Solution. Begin with the core claim above, use the worked case as the concrete example, state the condition highlighted by the warning, and choose a failure where that condition does not hold. A complete explanation connects all four pieces instead of listing them.

Mastery practice A. Reconstruct probability and simulation as long-run models from a blank page. Include the central definition, the governing rule, one required assumption, and one boundary case.

Solution. Start from the blue idea box, but rewrite it in my own words. Identify the objects and state, name the rule before using it, copy the relevant assumption from the details audit, and choose a boundary where that assumption becomes equality or fails. A complete solution explains why each part belongs; a list of vocabulary is not enough.

Mastery practice B. A classmate gets a polished-looking answer to a probability and simulation as long-run models problem but never states a model or checks the result. Write the shortest useful review of that solution.

Solution. I would ask four concrete questions: What exactly are the inputs and desired output? Which rule licenses the main step? Which assumption could invalidate it? Which independent check supports the conclusion? If the answer cannot survive those four questions, I treat the numerical or verbal result as unverified.

CHAPTER 6 Sampling distributions and standard error

The idea

A statistic varies across random samples. Its sampling distribution supplies bias, variability, and shape; standard error measures typical sample-to-sample movement, and larger independent samples usually reduce it at a square-root rate.

Rebuild the chapter from first principles

The claim I need to own is this: A statistic varies across random samples. Its sampling distribution supplies bias, variability, and shape; standard error measures typical sample-to-sample movement, and larger independent samples usually reduce it at a square-root rate. I do not count that as learned just because the sentence looks familiar. I should be able to name the objects, state what changes and what stays fixed, recover the rule from the definitions, and explain why the result is allowed.

The move

My working sequence

1. **Translate.** Rewrite the prompt in the language of sampling distributions and standard error. List the known quantities, the unknown, the units or types, and any hidden constraint.
2. **Choose.** State the theorem, model, algorithm, accounting rule, or physical law before substituting values. This is where I make the assumption visible.
3. **Execute.** Work one justified step at a time and keep intermediate state readable enough that I can audit it later.
4. **Verify.** Check a boundary case, sign, unit, invariant, order of magnitude, or alternative derivation. Then translate the result back into an ordinary sentence.

Details I would refuse to skip

- **Definitions:** I should be able to define every technical noun in the chapter without using that same noun in the definition.
- **Assumptions:** I write the condition that makes the method legal beside the method itself. A correct procedure under the wrong assumptions is still a wrong answer.
- **Representation:** I move freely among words, equations or code, a diagram, and a small numerical example. If one representation disagrees with another, that disagreement is useful evidence.
- **Failure modes:** I keep the nearest counterexample in mind: the boundary where the rule changes, the invariant breaks, or the model stops matching reality.
- **Communication:** My final answer names what the result means and what it does not establish. I do not let notation do the explanatory work for me.

Sanity check

Closed-note check. Can I teach sampling distributions and standard error on a blank page, derive or reconstruct its main rule, work a fresh example, and diagnose a deliberately broken solution? If I can only recognize the finished work, I am not done.

How this chapter connects

I read this chapter as the bridge from probability and simulation as long-run models to confidence intervals and significance tests. The earlier material supplies the language and constraints; this chapter adds a new reasoning move; the later material asks me to use that move without being told its name. That connection matters because exams and real projects mix chapters on purpose.

The full working model

- **Core claim.** A statistic varies across random samples.
- **What moves.** Its sampling distribution supplies bias, variability, and shape; standard error measures typical sample-to-sample movement, and larger independent samples usually reduce it at a square-root rate.
- **What keeps the answer honest.** The model becomes useful only when I can apply sampling distributions and standard error to an unfamiliar case and defend the assumptions.
- **Mastery standard.** I can define the objects, rebuild the rule, work a fresh case, compare alternatives, and diagnose a broken solution without relying on recognition.

How I actually think about it

The raw data vary within one sample; the standard error describes how my estimate would vary across new samples. Those are different kinds of spread.

The easy version

A statistic varies across random samples.

Worked example

Question. If a mean has standard error 6 at $n=25$, what is the approximate standard error at $n=100$ under the same conditions?

Walkthrough. Quadrupling the sample size halves the standard error because it scales as one over square root n . The new standard error is about 3.

Common miss

The fastest way to get this chapter wrong is to use the visible procedure without naming its assumption, units, time period, reference group, or boundary. I put that condition beside the work and verify the result independently.

Solved chapter practice

Practice 1. If a mean has standard error 6 at $n=25$, what is the approximate standard error at $n=100$ under the same conditions?

Solution. Quadrupling the sample size halves the standard error because it scales as one over square root n . The new standard error is about 3.

Practice 2. Explain sampling distributions and standard error to a classmate using one definition, one concrete example, one assumption, and one failure mode.

Solution. Begin with the core claim above, use the worked case as the concrete example, state the condition highlighted by the warning, and choose a failure where that condition does not hold. A complete explanation connects all four pieces instead of listing them.

Mastery practice A. Reconstruct sampling distributions and standard error from a blank page. Include the central definition, the governing rule, one required assumption, and one boundary case.

Solution. Start from the blue idea box, but rewrite it in my own words. Identify the objects and state, name the rule before using it, copy the relevant assumption from the details audit, and choose a boundary where that assumption becomes equality or fails. A complete solution explains why each part belongs; a list of vocabulary is not enough.

Mastery practice B. A classmate gets a polished-looking answer to a sampling distributions and standard error problem but never states a model or checks the result. Write the shortest useful review of that solution.

Solution. I would ask four concrete questions: What exactly are the inputs and desired output? Which rule licenses the main step? Which assumption could invalidate it? Which independent check supports the conclusion? If the answer cannot survive those four questions, I treat the numerical or verbal result as unverified.

CHAPTER 7 Confidence intervals and significance tests

The idea

A confidence interval combines an estimate with a margin of error under a repeated-sampling procedure. A significance test compares data with a null model through a statistic and p-value; statistical significance is not effect size, certainty, or practical importance.

Rebuild the chapter from first principles

The claim I need to own is this: A confidence interval combines an estimate with a margin of error under a repeated-sampling procedure. A significance test compares data with a null model through a statistic and p-value; statistical significance is not effect size, certainty, or practical importance. I do not count that as learned just because the sentence looks familiar. I should be able to name the objects, state what changes and what stays fixed, recover the rule from the definitions, and explain why the result is allowed.

The move

My working sequence

1. **Translate.** Rewrite the prompt in the language of confidence intervals and significance tests. List the known quantities, the unknown, the units or types, and any hidden constraint.
2. **Choose.** State the theorem, model, algorithm, accounting rule, or physical law before substituting values. This is where I make the assumption visible.
3. **Execute.** Work one justified step at a time and keep intermediate state readable enough that I can audit it later.
4. **Verify.** Check a boundary case, sign, unit, invariant, order of magnitude, or alternative derivation. Then translate the result back into an ordinary sentence.

Details I would refuse to skip

- **Definitions:** I should be able to define every technical noun in the chapter without using that same noun in the definition.
- **Assumptions:** I write the condition that makes the method legal beside the method itself. A correct procedure under the wrong assumptions is still a wrong answer.
- **Representation:** I move freely among words, equations or code, a diagram, and a small numerical example. If one representation disagrees with another, that disagreement is useful evidence.
- **Failure modes:** I keep the nearest counterexample in mind: the boundary where the rule changes, the invariant breaks, or the model stops matching reality.
- **Communication:** My final answer names what the result means and what it does not establish. I do not let notation do the explanatory work for me.

Sanity check

Closed-note check. Can I teach confidence intervals and significance tests on a blank page, derive or reconstruct its main rule, work a fresh example, and diagnose a deliberately broken solution? If I can only recognize the finished work, I am not done.

How this chapter connects

I read this chapter as the bridge from sampling distributions and standard error to simple regression, residuals, and responsible communication. The earlier material supplies the language and constraints; this chapter adds a new reasoning move; the later material asks me to use that move without being told its name. That connection matters because exams and real projects mix chapters on purpose.

The full working model

- **Core claim.** A confidence interval combines an estimate with a margin of error under a repeated-sampling procedure.
- **What moves.** A significance test compares data with a null model through a statistic and p-value; statistical significance is not effect size, certainty, or practical importance.

- **What keeps the answer honest.** The model becomes useful only when I can apply confidence intervals and significance tests to an unfamiliar case and defend the assumptions.
- **Mastery standard.** I can define the objects, rebuild the rule, work a fresh case, compare alternatives, and diagnose a broken solution without relying on recognition.

How I actually think about it

I report the estimated size and uncertainty before the p-value so the scientific question does not collapse into a yes-or-no ritual.

The easy version

A confidence interval combines an estimate with a margin of error under a repeated-sampling procedure.

Worked example

Question. A 95 percent interval for a mean difference is 1.2 to 4.8. What can be said?

Walkthrough. The procedure supports positive differences compatible with 1.2 to 4.8 under its assumptions. Zero is excluded, so the corresponding two-sided 5 percent test rejects, but practical importance still depends on context.

Common miss

The fastest way to get this chapter wrong is to use the visible procedure without naming its assumption, units, time period, reference group, or boundary. I put that condition beside the work and verify the result independently.

Solved chapter practice

Practice 1. A 95 percent interval for a mean difference is 1.2 to 4.8. What can be said?

Solution. The procedure supports positive differences compatible with 1.2 to 4.8 under its assumptions. Zero is excluded, so the corresponding two-sided 5 percent test rejects, but practical importance still depends on context.

Practice 2. Explain confidence intervals and significance tests to a classmate using one definition, one concrete example, one assumption, and one failure mode.

Solution. Begin with the core claim above, use the worked case as the concrete example, state the condition highlighted by the warning, and choose a failure where that condition does not hold. A complete explanation connects all four pieces instead of listing them.

Mastery practice A. Reconstruct confidence intervals and significance tests from a blank page. Include the central definition, the governing rule, one required assumption, and one boundary case.

Solution. Start from the blue idea box, but rewrite it in my own words. Identify the objects and state, name the rule before using it, copy the relevant assumption from the details audit, and choose a boundary where that assumption becomes equality or fails. A complete solution explains why each part belongs; a list of vocabulary is not enough.

Mastery practice B. A classmate gets a polished-looking answer to a confidence intervals and

significance tests problem but never states a model or checks the result. Write the shortest useful review of that solution.

Solution. I would ask four concrete questions: What exactly are the inputs and desired output? Which rule licenses the main step? Which assumption could invalidate it? Which independent check supports the conclusion? If the answer cannot survive those four questions, I treat the numerical or verbal result as unverified.

CHAPTER 8 Simple regression, residuals, and responsible communication

The idea

Least-squares regression summarizes a linear conditional mean with slope, intercept, fitted values, and residuals. Residual plots test structure; extrapolation, leverage, omitted variables, and subgroup differences limit interpretation.

Rebuild the chapter from first principles

The claim I need to own is this: Least-squares regression summarizes a linear conditional mean with slope, intercept, fitted values, and residuals. Residual plots test structure; extrapolation, leverage, omitted variables, and subgroup differences limit interpretation. I do not count that as learned just because the sentence looks familiar. I should be able to name the objects, state what changes and what stays fixed, recover the rule from the definitions, and explain why the result is allowed.

The move

My working sequence

1. **Translate.** Rewrite the prompt in the language of simple regression, residuals, and responsible communication. List the known quantities, the unknown, the units or types, and any hidden constraint.
2. **Choose.** State the theorem, model, algorithm, accounting rule, or physical law before substituting values. This is where I make the assumption visible.
3. **Execute.** Work one justified step at a time and keep intermediate state readable enough that I can audit it later.
4. **Verify.** Check a boundary case, sign, unit, invariant, order of magnitude, or alternative derivation. Then translate the result back into an ordinary sentence.

Details I would refuse to skip

- **Definitions:** I should be able to define every technical noun in the chapter without using that same noun in the definition.
- **Assumptions:** I write the condition that makes the method legal beside the method itself. A correct procedure under the wrong assumptions is still a wrong answer.

- **Representation:** I move freely among words, equations or code, a diagram, and a small numerical example. If one representation disagrees with another, that disagreement is useful evidence.
- **Failure modes:** I keep the nearest counterexample in mind: the boundary where the rule changes, the invariant breaks, or the model stops matching reality.
- **Communication:** My final answer names what the result means and what it does not establish. I do not let notation do the explanatory work for me.

Sanity check

Closed-note check. Can I teach simple regression, residuals, and responsible communication on a blank page, derive or reconstruct its main rule, work a fresh example, and diagnose a deliberately broken solution? If I can only recognize the finished work, I am not done.

How this chapter connects

I read this chapter as the bridge from confidence intervals and significance tests to the cumulative course model. The earlier material supplies the language and constraints; this chapter adds a new reasoning move; the later material asks me to use that move without being told its name. That connection matters because exams and real projects mix chapters on purpose.

The full working model

- **Core claim.** Least-squares regression summarizes a linear conditional mean with slope, intercept, fitted values, and residuals.
- **What moves.** Residual plots test structure; extrapolation, leverage, omitted variables, and subgroup differences limit interpretation.
- **What keeps the answer honest.** The model becomes useful only when I can apply simple regression, residuals, and responsible communication to an unfamiliar case and defend the assumptions.
- **Mastery standard.** I can define the objects, rebuild the rule, work a fresh case, compare alternatives, and diagnose a broken solution without relying on recognition.

How I actually think about it

My shorthand for this chapter is: my final paragraph says what was estimated, how uncertain it is, which design supports the claim, and what the model cannot establish.

The easy version

Least-squares regression summarizes a linear conditional mean with slope, intercept, fitted values, and residuals.

Worked example

Question. A fitted slope is 2.4 points per study hour. Interpret it without claiming causation.
Walkthrough. Within the observed range, students who differ by one study hour are predicted to differ by 2.4 points on average. The observational regression alone does not show that adding

an hour causes a 2.4-point increase.

Common miss

The fastest way to get this chapter wrong is to use the visible procedure without naming its assumption, units, time period, reference group, or boundary. I put that condition beside the work and verify the result independently.

Solved chapter practice

Practice 1. A fitted slope is 2.4 points per study hour. Interpret it without claiming causation.

Solution. Within the observed range, students who differ by one study hour are predicted to differ by 2.4 points on average. The observational regression alone does not show that adding an hour causes a 2.4-point increase.

Practice 2. Explain simple regression, residuals, and responsible communication to a classmate using one definition, one concrete example, one assumption, and one failure mode.

Solution. Begin with the core claim above, use the worked case as the concrete example, state the condition highlighted by the warning, and choose a failure where that condition does not hold. A complete explanation connects all four pieces instead of listing them.

Mastery practice A. Reconstruct simple regression, residuals, and responsible communication from a blank page. Include the central definition, the governing rule, one required assumption, and one boundary case.

Solution. Start from the blue idea box, but rewrite it in my own words. Identify the objects and state, name the rule before using it, copy the relevant assumption from the details audit, and choose a boundary where that assumption becomes equality or fails. A complete solution explains why each part belongs; a list of vocabulary is not enough.

Mastery practice B. A classmate gets a polished-looking answer to a simple regression, residuals, and responsible communication problem but never states a model or checks the result. Write the shortest useful review of that solution.

Solution. I would ask four concrete questions: What exactly are the inputs and desired output? Which rule licenses the main step? Which assumption could invalidate it? Which independent check supports the conclusion? If the answer cannot survive those four questions, I treat the numerical or verbal result as unverified.

Cumulative study plan

I use this packet in three passes. First I reconstruct each model and work its examples without looking. Second I mix chapters so the tool is no longer named for me. Third I explain a complete case aloud, including assumptions, units, uncertainty, failure modes, and what evidence would change my conclusion.